

Clearness Index–Based Computation And Evaluation Of Mean Daily Insolation And Optimal Fixed Tilt Angle For PV Installation In Uyo, Akwa Ibom State

Anyanime Tim Umoette¹

Department of Electrical and Electronic Engineering,
Akwa Ibom State University, Nigeria

Silas Abraham Friday²

Department of Electrical/ Electronic Engineering
Adekeke University, Ede Osun State
silas.abraham@adelekeuniversity.edu.ng

Kingsley Bassey Clement³

Department of Electrical/ Electronic Engineering
University of Uyo, Akwa Ibom State

Abstract— In this work, clearness index–based computation and evaluation of mean daily insolation and optimal fixed tilt angle for PV installation in Uyo, Akwa Ibom State is presented. Specifically, this study is concerned with presenting an approach for, one computing the daily mean insolation on a tilted PV plane and two, obtaining the yearly fixed optimal PV tilt angle for the PV installation based on the computed daily mean insolation. The mathematical expressions and algorithm used are presented in 12 steps. Sample computation was first performed for tilt angles ranging from 1° to 70° and the optimal tilt angle obtained is 2.998468°. A second computation was performed based on tilt angles ranging from 1° to 10° and the optimal tilt angle obtained is 5.643515°. The results showed that the optimal tilt angle of 5.643515° has the highest annual mean transposition ratio of 1.0377041, followed by optimal tilt angle of 5.041299° which has the annual mean transposition ratio of 1.0376606 and error of about 0.004%. The initial estimate of the optimal tilt angle is 2.998468° and it has error in the annual mean transposition ratio of 0.081 %. Also, the optimal tilt angle obtained is compared with four other empirical optimal tilt angle options. The results showed that two of the available options have errors less than 0.01 % of the optimal solution while the other remaining two options have error above 4 % of the optimal solution.

Keywords— Clearness Index, Mean Daily Insolation, Fixed Optimal Tilt Angle, Transposition Factor

1. INTRODUCTION

Nowadays, solar power system has dominated the alternative energy solution in Nigeria [1,2,3]. This is because of the abundant supply of solar radiation all through the year in every location in Nigeria [4,5,6]. Also, the continuous rise in the cost of grid energy supply and the rise in fossil fuel cost also drives the rise in the adoption of solar energy option [7,8,9].

In any case, design and installation of solar power plant require knowledge of the solar radiation available at the installation site [10,11]. In most case, mean daily insolation and annual mean of the daily insolation values are used in the design [12,13]. In addition, photovoltaic (PV) module is usually tilted to capture the most of the solar radiation available at a given location [14,15,16]. The knowledge of the solar radiation on the horizontal surface and the optimal tilt angle for any given location can greatly help in the design of solar power with enhanced energy yield. Accordingly, in this work, clearness index–based method for computation of the mean daily insolation, the annual mean of the daily insolation and the annual fixed optimal tilt angle for the PV power plant is presented. Presently, there are some available empirical expressions for determining optimal tilt angle based on location latitude [17,18,19]. In this work, about 4 of such empirical optimal angle options and evaluated in respect of the optimal tilt angle obtained in this work.

2. METHODOLOGY

This study is concerned with presenting an approach for, one computing the daily mean insolation on a tilted PV plane and two, obtaining the yearly fixed optimal PV tilt angle for the PV installation based on the computed daily mean insolation. In order to compute the daily mean

insolation on a tilted PV plane, the following parameters are needed;

$$\beta_{tilt} = Lat + 15 \quad (3)$$

$$\beta_{tilt} = 0.69 (Lat) + 3.7 \quad (4)$$

- i. Solar constant, G_{sc} where $G_{sc} = 1353 \text{ W/m}^2$
- ii. The PV module installation site latitude (Lat) and longitude (Lon). In this study Lat = 5.041299 and Lon = 7.974398 which is the location for University of Uyo Permanent site.
- iii. The PV module tilt angle (β_{tilt}). Each of the following optimal tilt angle options are considered and compare with the optimum tilt angle obtained in this study [20,21,22,23];

$$\beta_{tilt} = Lat \quad (1)$$

$$\beta_{tilt} = Lat + 10 \quad (2)$$

- iv. The ground reflectance parameter (ρ). A value of $\rho = 0.2$ is used
- v. The computation is performed for each of the 365 days in a year. Hence, the day number, $n = 1, 2, 3, \dots, 365$.
- vi. The index ($KT_{(n)}$); The data for $KT_{(n)}$ for each of the 365 days in a year are obtained from NASA portal based on the PV module installation site latitude (Lat) and longitude (Lon). The scatter plot of the KT dataset is shown in Figure 1.

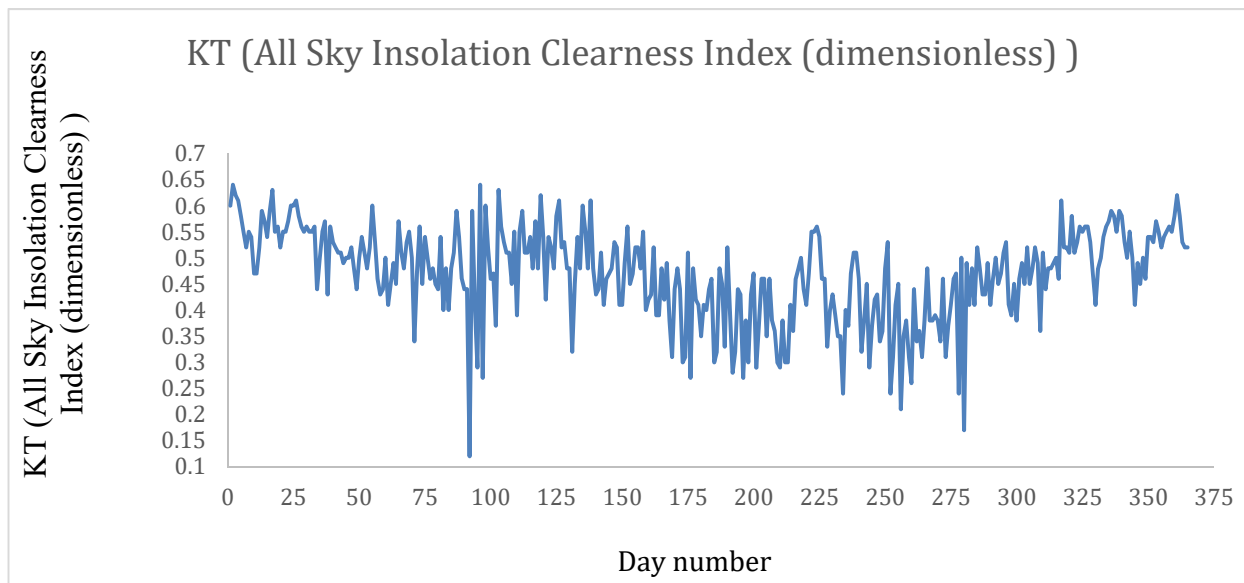


Figure 1 The scatter plot of the KT dataset

The mathematical expressions and algorithm used to compute the daily mean insolation on a tilted PV plane and to obtain the yearly fixed optimal PV tilt angle for the PV installation are presented in the following 12 steps [24];

Step 1: Calculate the declination angle, δ_n where;

$$\delta_{(n)} = 23.45 \text{ Sin} \left(\frac{360 \times (284 + n)}{365} \right) \quad (5)$$

Step 2: Calculate the sunset hour angle, $w_{ss(n)}$ where;

$$w_{ss(n)} = \cos^{-1} \left(-\tan(\delta_{(n)}) \tan(Lat) \right) \quad (6)$$

Step 3: Calculate the extraterrestrial insolation on a horizontal surface, $\bar{H}_{o(n)}$ where;

$$\bar{H}_{o(n)} = (K_{o1(n)}) (K_{o2(n)}) \quad (7)$$

Where;

$$K_{o1(n)} = \left(\frac{24}{\pi} \right) (3600)(G_{sc}) \left(1 + 0.033 \text{ Cos} \left(\frac{360(n)}{365} \right) \right) \quad (8)$$

$$K_{o2(n)} = \left(\text{Cos}(Lat) \text{ Cos}(\delta_{(n)}) \text{ Sin}(w_{ss(n)}) \right) + \left(\left(\frac{2(\pi)(w_{ss(n)})}{360} \right) \text{ Sin}(Lat) \text{ Sin}(\delta_{(n)}) \right) \quad (9)$$

Step 4: Calculate the terrestrial insolation on a horizontal surface, $\bar{H}_{(n)}$ where;

$$\bar{H}_{(n)} = (\bar{H}_{o(n)}) (KT_{(n)}) \quad (10)$$

Step 5: Calculate the diffuse radiation on a horizontal surface, $\bar{H}_{d(n)}$ where;

$$\frac{\bar{H}_{d(n)}}{\bar{H}_{(n)}} = K_{d1(n)} - K_{d2(n)} \quad (11)$$

Where;

$$K_{d1(n)} = 0.775 + 0.00653 (w_{ss(n)} - 90) \quad (12)$$

$$K_{d2(n)} = (0.505 + 0.00455 (w_{ss(n)} - 90)) (\text{Cos}(115 (KT_{(n)}) - 103)) \quad (13)$$

$$\bar{H}_{d(n)} = \left(\frac{\bar{H}_{d(n)}}{\bar{H}_{(n)}} \right) (\bar{H}_{(n)}) \quad (14)$$

Step 6: Calculate $Lat\beta$ where;

$$Lat\beta = \begin{cases} Lat - \beta & \text{if } Lat \geq 0 \text{ (Northern hemisphere)} \\ Lat + \beta & \text{if } Lat < 0 \text{ (Southern hemisphere)} \end{cases} \quad (15)$$

Step 7: Calculate the sunset hour angle for the titled collector, $w_{sT(n)}$ where;

$$w_{sT(n)} = \text{minimum} \left(\begin{array}{l} \left[\cos^{-1} \left(-\tan(\delta_{(n)}) \tan(\text{Lat}) \right) \right] \\ \left[\cos^{-1} \left(-\tan(\delta_{(n)}) \tan(\text{Lat}\beta) \right) \right] \end{array} \right) \quad (16)$$

Step 8: Calculate the ratio of the daily mean beam radiation on the tilted surface to the daily mean beam radiation on a horizontal plane, $\bar{R}_{b(n)}$ where;

$$\bar{R}_{b(n)} = \frac{(K_{Rb1(n)}) + (K_{Rb2(n)})}{(K_{Rb3(n)}) + (K_{Rb4(n)})} \quad (17)$$

Where;

$$K_{Rb1(n)} = \left(\cos(\text{Lat}\beta) \cos(\delta_{(n)}) \sin(w_{sT(n)}) \right) \quad (18)$$

$$K_{Rb2(n)} = \left(\frac{(\pi)(w_{sT(n)})}{180} \right) \sin(\text{Lat}\beta) \sin(\delta_{(n)}) \quad (19)$$

$$K_{Rb3(n)} = \left(\cos(\text{Lat}) \cos(\delta_{(n)}) \sin(w_{ss(n)}) \right) \quad (20)$$

$$K_{Rb4(n)} = \left(\frac{(\pi)(w_{ss(n)})}{180} \right) \sin(\text{Lat}) \sin(\delta_{(n)}) \quad (21)$$

Step 9: Calculate the mean daily total radiation on the tilted surface, $\bar{H}_{T(n)}$ where;

$$\bar{H}_{T(n)} = \left((\bar{H}_{(n)} - \bar{H}_{d(n)}) \bar{R}_{b(n)} \right) + \left(\bar{H}_{d(n)} \left(\frac{1 - \cos(\beta)}{2} \right) \right) + \left(\rho \left(\frac{1 - \cos(\beta)}{2} \right) \right) \quad (22)$$

Step 10: Calculate the transposition ratio, $\text{Tr}_{\beta}(n)$ for tilt angle, β where;

$$\text{Tr}_{\beta}(n) = \frac{\bar{H}_{T(n)}}{\bar{H}_{T(n)} \beta=0^\circ} = \frac{\bar{H}_{T(n)}}{\bar{H}_{(n)}} \quad (23)$$

Step 11: Calculate the annual mean transposition ratio, AMTr_{β} for tilt angle, β where;

$$\text{AMTr}_{\beta} = \frac{1}{365} \left[\sum_{n=1}^{365} \left(\frac{\bar{H}_{T(n)}}{\bar{H}_{T(n)} \beta=0^\circ} \right) \right] = \frac{1}{365} \left[\sum_{n=1}^{365} \left(\frac{\bar{H}_{T(n)}}{\bar{H}_{(n)}} \right) \right] \quad (24)$$

Step 12: Plot the graph of AMTr_{β} versus β and fit quadratic trend line to the graph. Determine the first derivative of the quadratic expression, $\frac{\delta(\text{AMTr}_{\beta})}{\delta(\beta)}$, set the derivative to zero and solve for the value of β at $\frac{\delta(\text{AMTr}_{\beta})}{\delta(\beta)} = 0$. The value of β obtained is the optimal tilt angle.

3 RESULTS AND DISCUSSION

Based on the mathematical expressions and 12 steps algorithm, sample computation of the daily mean insolation on a tilted PV plane is conducted for a case study PV installation located at University of Uyo Permanent site with latitude of 5.041299 and longitude of 9.041299. The results for the sample calculation at University of Uyo Permanent site are presented for each of the 12 steps in the algorithm.

Step 1: For January 1st 2023, $n = 1$, hence, the declination angle, δ_n is given as;

$$\delta_{(n)} = 23.45 \sin \left(\frac{360 \times (284 + 1)}{365} \right) = -23.01163673^\circ$$

Step 2: The latitude = 5.041299 and longitude = 7.17849631, hence, the sunset hour angle, $w_{ss(n)}$ is given as;

$$w_{ss(n)} = \cos^{-1} \left(-\tan(-23.0116367) \tan(5.041299) \right) = 87.85284^\circ$$

Step 3: Given that $(G_{sc}) = 1353$ (kWh/m².day) for January 1, then, the extraterrestrial insolation on a horizontal surface, $\bar{H}_{o(n)}$ is given as;

$$K_{o1(n)} = \left(\frac{24}{\pi} \right) (3600)(1353) \left(1 + 0.033 \cos \left(\frac{360(1)}{365} \right) \right) = 38.4379248$$

$$K_{o2(n)} = \left(\cos(5.041299) \cos(-23.0116367) \sin(87.85284) \right) + \left(\left(\frac{2(\pi)(87.85284)}{360} \right) \sin(5.041299) \sin(-23.0116367) \right) = 0.86354937$$

$$\bar{H}_{o(n)} = (38.4379248)(0.86354937) = 33.1930458$$

Step 4: Given that $KT_{(n)} = 0.6$, then, the terrestrial insolation on a horizontal surface, $\bar{H}_{(n)}$ is given as;

$$\bar{H}_{(n)} = (33.1930458)(0.6) = 19.91582751$$

Step 5: Calculate the diffuse radiation on a horizontal surface, $\bar{H}_{d(n)}$ where;

$$K_{d1(n)} = 0.775 + 0.00653 (87.85284 - 90) = 0.740979$$

$$K_{d2(n)} = \left(0.505 + 0.00455 (87.85284 - 90) \right) \left(\cos(115(0.6) - 103) \right) = 0.410565$$

$$\frac{\bar{H}_{d(n)}}{\bar{H}_{(n)}} = 0.740979 - 0.410565 = 0.330414$$

$$\bar{H}_{d(n)} = (0.330414)(19.91582751) = 6.580477$$

Step 6: The tilt angle, $\beta = 7.17849631$, and $\text{Lat} = 5.041299 > 0$, hence, $\text{Lat}\beta$ where;

$$\text{Lat}\beta = 5.041299 - 7.17849631 = -2.13719731$$

Step 7: The sunset hour angle for the titled collector, $w_{sT(n)}$ where;

$$w_{sT(n)} = \text{minimum} \left(\begin{array}{l} \left[\cos^{-1} \left(-\tan(-23.01163673) \tan(5.041299) \right) \right], \\ \left[\cos^{-1} \left(-\tan(-23.01163673) \tan(-2.13719731) \right) \right] \end{array} \right) = 87.85284$$

Step 8: The ratio of the daily mean beam radiation on the tilted surface to the daily mean beam radiation on a horizontal plane, $\bar{R}_{b(n)}$ is given as;

$$\begin{aligned}
 &K_{Rb1(n)} \\
 &= (\text{Cos}(-2.13719731) \text{Cos}(-23.01163673) \text{Sin}(87.85284)) \\
 &= 0.919139438 \\
 &K_{Rb2(n)} = \\
 &\left(\frac{(\pi)(87.85284)}{180}\right) \text{Sin}(-2.13719731) \text{Sin}(-23.01163673) = \\
 &0.916221184 \\
 &K_{Rb3(n)} = \\
 &(\text{Cos}(5.041299) \text{Cos}(-23.01163673) \text{Sin}(87.85284)) = \\
 &0.022353226 \\
 &K_{Rb4(n)} = \\
 &\left(\frac{(\pi)(87.85284)}{180}\right) \text{Sin}(5.041299) \text{Sin}(-23.01163673) = \\
 &-0.05267181 \\
 &\bar{R}_b(n) = \frac{(0.919139438)+(0.916221184)}{(0.022353226)+(-0.05267181)} = \frac{0.941492664}{0.863549374} = \\
 &1.090259217
 \end{aligned}$$

Step 9: Now, $\rho = 0.2$, then, the mean daily total radiation on the tilted surface, $\bar{H}_{T(n)}$ is given as;

$$\begin{aligned}
 \bar{H}_{T(n)} &= ((19.91582751 - 6.580477)(1.090259217)) \\
 &+ \left(6.580477 \left(\frac{1 - \text{Cos}(7.1784963)}{2}\right)\right) \\
 &+ \left(0.2 \left(\frac{1 - \text{Cos}(7.1784963)}{2}\right)\right) \\
 &= 21.09445969
 \end{aligned}$$

$$\begin{aligned}
 \bar{H}_{T(n)} &= 14.53898916 + 6.5546867 + 0.000783833 \\
 &= 21.09445969
 \end{aligned}$$

Step 10: The transposition ratio, $\text{Tr}_\beta(n)$ for tilt angle, β is given as;

$$\text{Tr}_\beta(n) = \frac{21.09445969}{19.91582751} = 1.059180678$$

Step 11: The annual mean transposition ratio, AMTr_β for tilt angle, β is given as;

$$\text{AMTr}_\beta = \frac{1}{365} \left[\sum_{n=1}^{365} \left(\frac{\bar{H}_{T(n)}}{\bar{H}_{T(n)} \beta=0^\circ} \right) \right] = 1.0374224$$

Step 12: The graph of $\bar{H}_{T(n)}$ and $\bar{H}_{T(n)} \beta=0^\circ$ (that is $\bar{H}_{(n)}$) versus day number for 365 days are presented in Figure 2 while the graph of the transposition ratio, $\text{Tr}_\beta(n)$ for the tilt angle, $\beta = 7.17849631$ is shown in Figure 3.

The results of the PV tilt Angle, β ($^\circ$) and the corresponding the annual mean transposition ratio obtained are presented in Table 2. The annual mean transposition ratio results are obtained from the daily transposition ratio computed for a whole year. The computation was performed for tilt angles ranging from 1° to 70° . Based on this range of tilt angle, the optimal tilt angle is obtained from the trend line equation (in Equation 25 and shown in Figure 4) and its derivative (in Equation 26) as 2.998468° .

$$\text{AMTr}_\beta = -0.0000979 \beta^2 + 0.0005871 \beta + 1.0375494 \quad (25)$$

$$\frac{d(\text{AMTr}_\beta)}{d\beta} = -0.000196 \beta + 0.0005871 \quad (26)$$

$$\beta = 2.998468$$

In order to obtain a more accurate optimal tilt angle, the range of the tilt angle considered is narrowed down from 1° to 10° . The computation was performed based on tilt angles ranging from 1° to 10° . Based on this range of tilt angle, the optimal tilt angle is obtained from the trend line equation (in Equation 27 and shown in Figure 5) and its derivative (in Equation 28) as 5.643515° .

$$\text{AMTr}_\beta = -0.0001195x^2 + 0.0013488x + 1.0338976 \quad (27)$$

$$\frac{d(\text{AMTr}_\beta)}{d\beta} = -0.000196 \beta + 0.0005871 \quad (28)$$

$$\beta = 5.643515$$

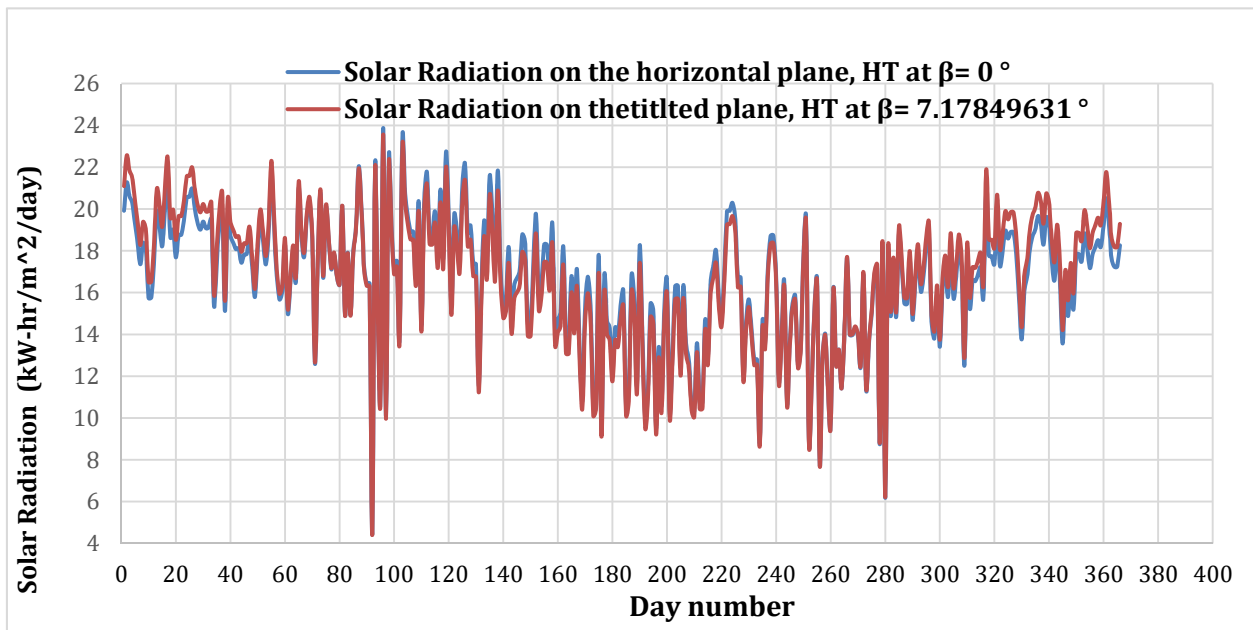


Figure 2 The graph of computed $\bar{H}_{T(n)}$ for the tilt angle, $\beta = 7.17849631$ and $\bar{H}_{T(n) \beta=0^\circ}$ (that is $\bar{H}_{(n)}$) versus day number for 365 days

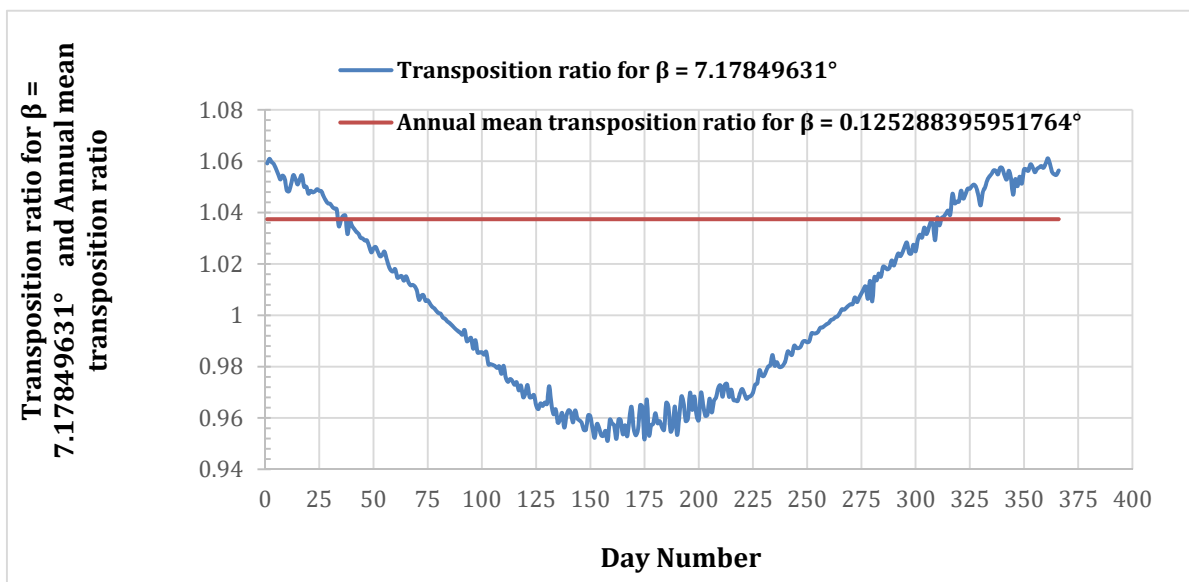
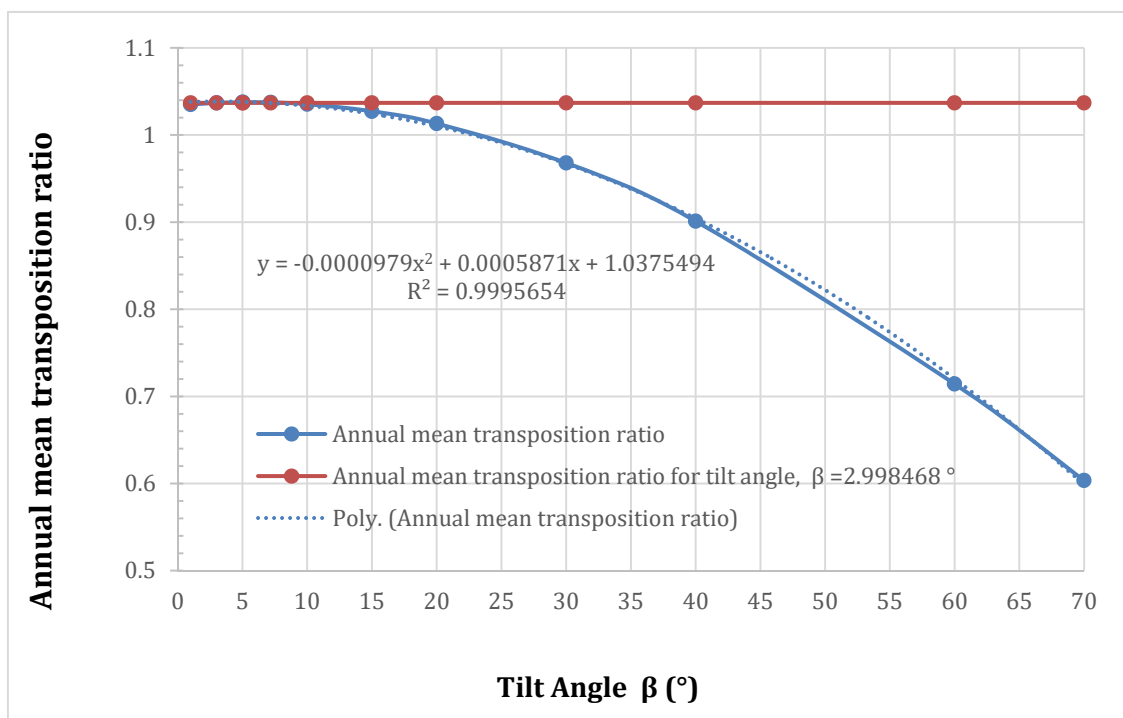


Figure 3 The graph of the transposition ratio, $Tr_\beta(n)$ versus day number for the tilt angle, $\beta = 7.17849631$

Table 1 The PV tilt Angle, β ($^{\circ}$) and the corresponding the annual mean transposition ratio

S/N	PV tilt Angle, β ($^{\circ}$)	Annual mean transposition ratio
1	1	1.035127
2	3	1.0368684
3	5	1.037654
4	5.041299	1.0376606
5	7.178496	1.0374224
6	10	1.035436
7	15	1.027262
8	20	1.013201
9	30	0.967948
10	40	0.90124
11	60	0.714288
12	70	0.60353
	Optimal tilt angle estimates	Annual mean transposition ratio at optimal tilt angle options
Initial approximation of the optimal tilt angle		2.998468
The optimal tilt angle		5.643515

Figure 4 The graph of the annual mean transposition ratio versus PV tilt angle, β for $1 \leq \beta \leq 70$

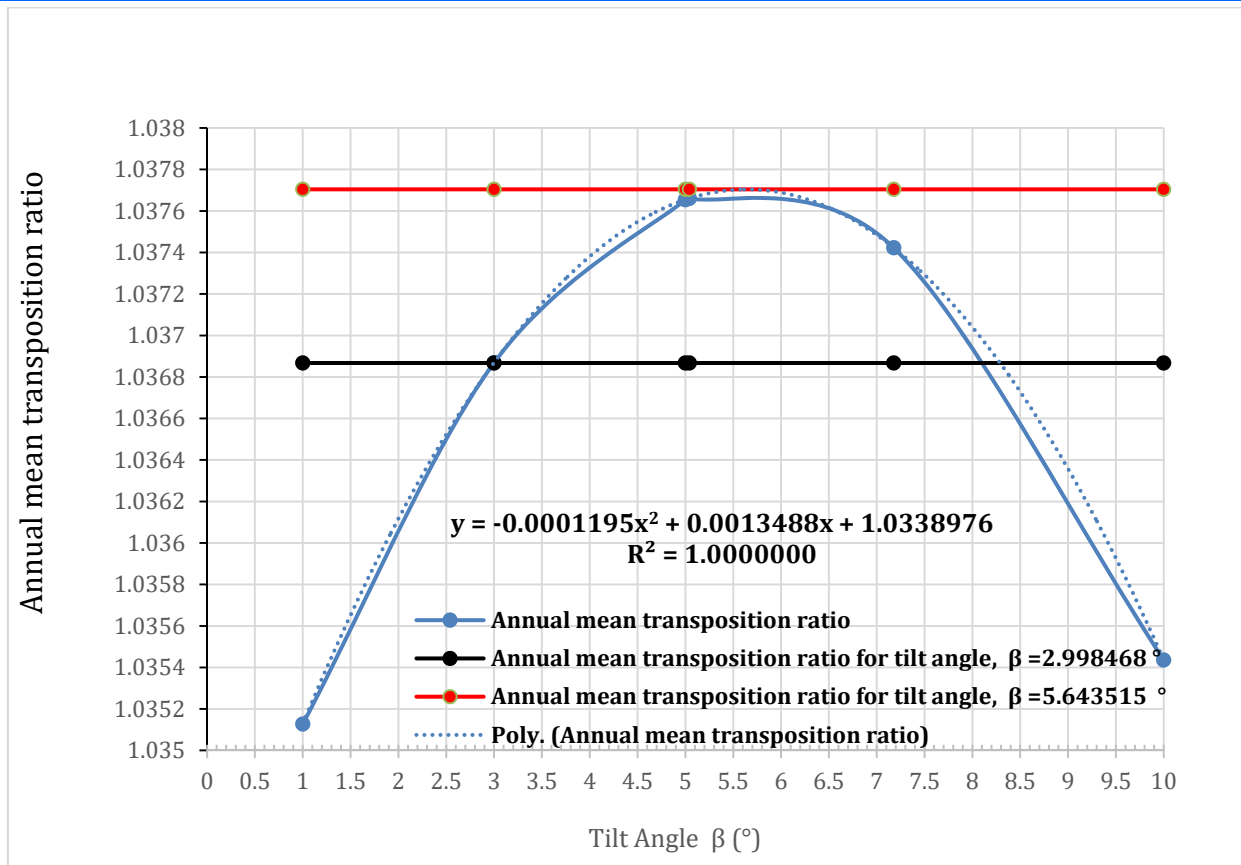


Figure 5 The graph of the annual mean transposition ratio versus PV tilt angle, β for $1 \leq \beta \leq 10$

Comparison of the results of the various optimal tilt angle options are presented in Table 2 and Figure 5 while Figure 6 presents the comparison of the error in the various optimal tilt angle options. It can be seen from Table 2, Figure 5 and Figure 6 that the optimal tilt angle of 5.643515° has the highest annual mean transposition ratio of 1.0377041, followed by optimal tilt angle of 5.041299° which has the annual mean transposition ratio of 1.0376606

and error of about 0.004%. The initial estimate of the optimal tilt angle is 2.998468° and it has error in the annual mean transposition ratio of 0.081 %, as shown in Figure 6. Generally, the results showed that the empirical optimal angle options in Equation 1 and Equation 4 can be adopted when error of about 0.01% can be accommodated. On the other hand, empirical optimal angle options in Equation 2 and Equation 3 gave errors above 4 %.

Table 2 Comparison of the results of the various optimal tilt angle options

	Optimal tilt angle options	Annual mean transposition ratio at optimal tilt angle options	
Initial approximation of the optimal tilt angle	2.998468	1.0368674	
The optimal tilt angle	5.643515	1.0377041	
$\beta_{tilt} = Lat$	5.041299	1.0376606	
$\beta_{tilt} = 0.69 (Lat) + 3.7$	7.178496	1.0374224	
$\beta_{tilt} = Lat + 10$	15.0413	0.993492	
$\beta_{tilt} = Lat + 15$	20.0413	0.979846	

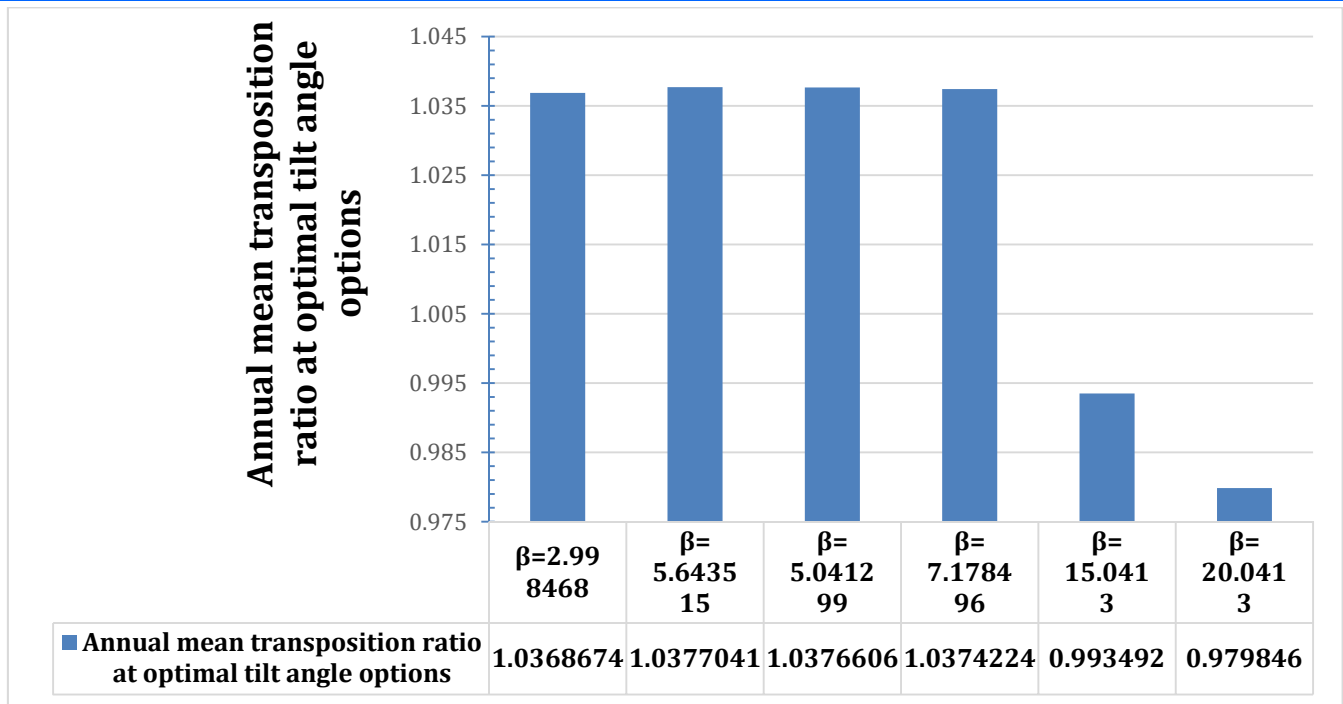


Figure 6 Comparison of the results of the various optimal tilt angle options

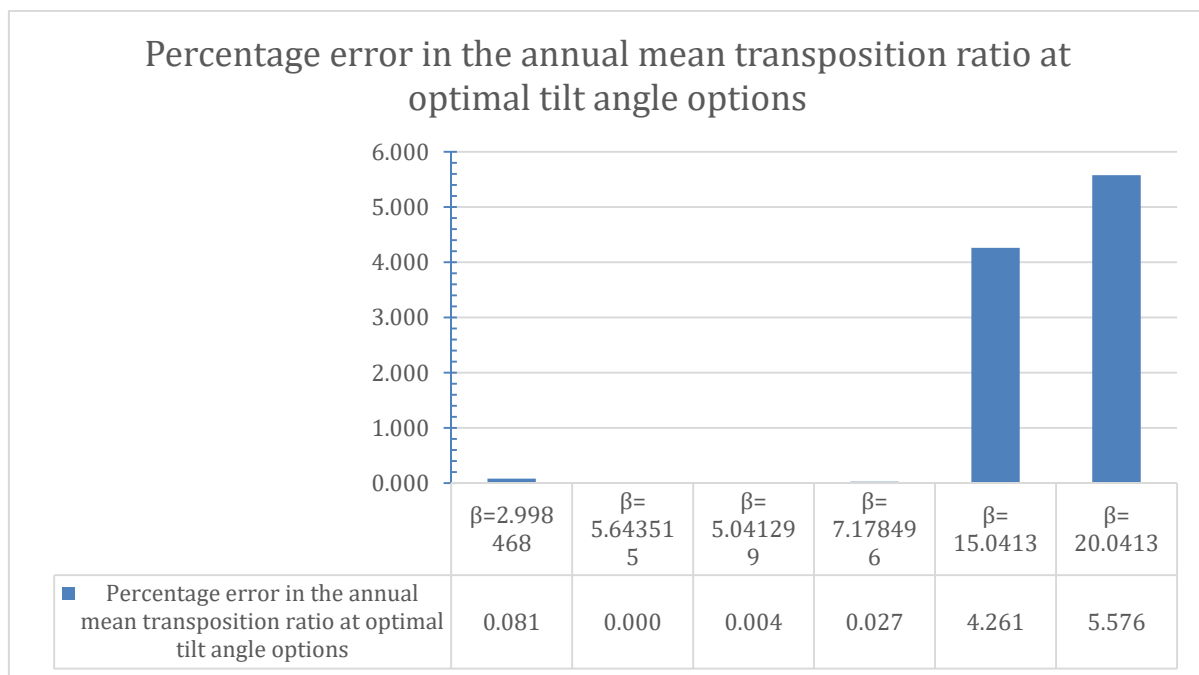


Figure 7 Comparison of the error in the various optimal tilt angle options

4. CONCLUSION

An the approach for using clearness index dataset and sun position angles in computing the daily mean insolation on a tilted PV plane and also in obtaining the yearly fixed optimal tilt angle for the PV installation are presented. The analytical expression and sequential steps for the computations are presented along with sample computation based on a case study PV installation at University of Uyo permanent site. The optimal tilt angle obtained is compared with four other empirical optimal tilt angle options. The

results showed that two of the available options have errors less than 0.01 % of the optimal solution while the other remaining two options have error above 4 % of the optimal solution.

REFERENCES

1. Agbo, E. P., Edet, C. O., Magu, T. O., Njok, A. O., Ekpo, C. M., & Louis, H. (2021). Solar energy: A panacea for the electricity generation crisis in Nigeria. *Heliyon*, 7(5).

2. **Umoren M. A.** and Udoakah, Y. N. (2015) Analysis of Solar Potential and Energy Production of Photovoltaic Modules at Optimum Tilt angles in Nigeria, *Nigerian Journal of Solar Energy*, vol. 26, PP.92-99
3. Owebor, K., Diemuodeke, E. O., Briggs, T. A., & Imran, M. (2021). Power Situation and renewable energy potentials in Nigeria—A case for integrated multi-generation technology. *Renewable Energy*, 177, 773-796.
4. Amadi, S. O., Dike, T., & Nwokolo, S. C. (2020). Global solar radiation characteristics at Calabar and Port Harcourt cities in Nigeria. *Trends in Renewable Energy*, 6(2), 111-130.
5. Ayodele, T. R., Ogunjuyigbe, A. S. O., & Nwakanma, K. C. (2021). Solar energy harvesting on building's rooftops: A case of a Nigeria cosmopolitan city. *Renewable Energy Focus*, 38, 57-70.
6. Jacob Iniobong Joseph, **Umoren Mfonobong Anthony**, and Idorenyin Markson (2016). Development of Optimal Site Selection Method for Large Scale Solar Photovoltaic Power Plant, *Mathematical and Software Engineering*, Vol. 2, No. 2 (2016), pp. 66-75
7. Nijse, F. J., Mercure, J. F., Ameli, N., Larosa, F., Kothari, S., Rickman, J., ... & Pollitt, H. (2023). The momentum of the solar energy transition. *Nature Communications*, 14(1), 6542.
8. Anyanime Tim Umoette, Mbetobong Udo Festus, Emmanuel A. Ubom. Development of Site Specific Optimal Tilt Angle Model for Fixed Tilted Plane PV Installation in Akwa Ibom State, Nigeria. *Science Journal of Energy Engineering*. Vol. 4, No. 6, 2016, pp. 50-55. DOI: 10.11648/j.sjee.20160406.11.
9. Bogdanov, D., Ram, M., Aghahosseini, A., Gulagi, A., Oyewo, A. S., Child, M., ... & Breyer, C. (2021). Low-cost renewable electricity as the key driver of the global energy transition towards sustainability. *Energy*, 227, 120467.
10. Aul, M. M., Enyia, J. D., & Lawal, T. (2022). Design and Installation of a 3.5 KVA Solar Photovoltaic Power System. *ATBU Journal of Science, Technology and Education*, 10(1), 319-330.
11. Anyanime Tim Umoette1, Simeon Ozuomba2*, Nseobong I. Okpura3. Comparative Analysis of the Solar Potential of Offshore and Onshore Photovoltaic Power System. *Mathematical and Software Engineering*, Vol. 3, No. 1 (2017), 124-138. Var psilon Ltd, varepsilon.com
12. DW, M., & Yakubu, D. (2011). Estimation of mean monthly global solar radiation in Yola-Nigeria using angstrom model.
13. Abdulsalam, D., Mbamali, I., Usman, M., & Bala, K. (2014). Insolation Levels Using Temperature Model for Sustainable Application of Photovoltaic Technology in Some Selected Locations of Nigeria. *Journal of Energy Technologies and Policy*, 4(1), 19-28.
14. Melhem, R., & Shaker, Y. (2023). Optimum tilt angle and solar radiation of photovoltaic modules for gulf collaboration council countries. *International Journal of Energy Research*, 2023.
15. Yunus Khan, T. M., Soudagar, M. E. M., Kanchan, M., Afzal, A., Banapurmath, N. R., Akram, N., ... & Shahapurkar, K. (2020). Optimum location and influence of tilt angle on performance of solar PV panels. *Journal of Thermal Analysis and Calorimetry*, 141, 511-532.
16. N'Tsoukpoe, K. E. (2022). Effect of orientation and tilt angles of solar collectors on their performance: Analysis of the relevance of general recommendations in the West and Central African context. *Scientific African*, 15, e01069.
17. Yadav, A. K., & Chandel, S. S. (2013). Tilt angle optimization to maximize incident solar radiation: A review. *Renewable and sustainable energy reviews*, 23, 503-513.
18. Kacira, M., Simsek, M., Babur, Y., & Demirkol, S. (2004). Determining optimum tilt angles and orientations of photovoltaic panels in Sanliurfa, Turkey. *Renewable energy*, 29(8), 1265-1275.
19. Khasawneh, Q. A., Damra, Q. A., & Bany Salman, O. H. (2015). Determining the Optimum Tilt Angle for Solar Applications in Northern Jordan. *Jordan Journal of Mechanical & Industrial Engineering*, 9(3).
20. González-González, E., Martín-Jiménez, J., Sánchez-Aparicio, M., Del Pozo, S., & Lagüela, S. (2022). Evaluating the standards for solar PV installations in the Iberian Peninsula: Analysis of tilt angles and determination of solar climate zones. *Sustainable Energy Technologies and Assessments*, 49, 101684.
21. Siraki, A. G., & Pillay, P. (2012). Study of optimum tilt angles for solar panels in different latitudes for urban applications. *Solar energy*, 86(6), 1920-1928.
22. Darhmaoui, H., & Lahjouji, D. (2013). Latitude based model for tilt angle optimization for solar collectors in the Mediterranean region. *Energy Procedia*, 42, 426-435.
23. Ajao, K. R., Ambali, R. M., & Mahmoud, M. O. (2013). Determination of the optimal tilt angle for solar photovoltaic panel in Ilorin, Nigeria. *Journal of Engineering Science and Technology Review*, 6(1), 87-90.
24. Bush, M. (1984). NOTES ON SOLAR ENERGY from the Training in Alternative Energy Technologies Program at the.